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Queuing delay models for single-lane roundabouts†

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This article presents an analytical model for a few important operational characteristics of single-lane roundabouts in the United States. In particular, we obtain renewal-based analytical expressions for the mean and variance of the time required for entry into the circulating stream for an arbitrary vehicle occupying the first position of the approach, regardless of the distribution of time headways for the circulating stream. These analytical models are subsequently applied in a M/G/1 queuing model to compute the steady-state average delay and length of the queue at the approach under stable conditions. The analytical models are validated by comparing numerical results for average delay with field observations obtained at six single-lane roundabout sites in the United States. The models are shown to perform well under a range of circulating stream flow rates.

Keywords: Roundabout operations; Roundabout delay; Headway distribution; Delay; Unsignalized operations

1. Introduction

In the United States, unsignalized intersections exist in various forms, such as T-intersections, two-way stop controlled intersections and all-way stop controlled intersections. An at-grade intersection known as the roundabout has been introduced in the United States. In the early 1940s, a similar intersection known as the traffic circle gained much popularity; however, roundabouts differ from traffic circles in that they include yield on entry, deflection on approaches and flared entries (Myers 1994). In fact, some publications consider roundabouts as a subset of traffic circles with greater restrictions on placement, design and operational characteristics (FDOT and MdSHA 1995).

In 1994, the first multi-lane roundabout freeway interchange in the United States opened in Vail, CO. Although the interchange has been successful to improve traffic conditions,

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roundabout installations at high volume intersections in the United States are rare. Many state and local officials are taking a conservative approach to the implementation of round-abouts and are limiting their use to single-lane sites until their suitability is better understood in the United States.

Despite the limited use of roundabouts in this country, many new roundabouts are being planned or designed by local and state engineers in the absence of a comprehensive study on the safety and operational performance of roundabouts. An operational method (based on the performance of American drivers) is required to allow practitioners to realistically compare various design alternatives. Typical operational performance assessment includes capacity prediction and various performance measure estimates. Capacity models are used to estimate the maximum hourly flow rate at which vehicles can reasonably expect to enter an intersection under prevailing conditions during a given time period. For a driver waiting to enter the circulating stream, measures for intersection analyses include estimates of average delay per vehicle (i.e. time in queue, time required for an acceptable gap and acceleration/deceleration time) and average queue length. The current US Highway Capacity Manual (HCM 2000) contains a procedure for estimating capacity at single-lane roundabout approaches based on gap acceptance procedures developed in Australia and limited United States field data. The current HCM does not contain a procedure for estimating delay at either single-lane or multi-lane roundabouts.

The main contributions of this article are as follows. First, we present analytical results for the mean and variance of the random time required for a vehicle waiting at the roundabout approach to enter the circulating stream under stable conditions. We term this quantity as the service time. Subsequently, the results of our analytical models may be applied in a steady-state queuing model to provide estimates for the average delay of vehicles on the roundabout approach as well as the average queue length. Finally, our analytical results for average delay are validated via field data obtained from several single-lane roundabout sites in the United States. The model is unique in that it is suited to handle any interarrival distribution whatsoever for the circulating traffic stream. To our knowledge, this general model of the mean and variance of service time is the first of its kind in that it explicitly models the service time apart from estimates of capacity. Limitations to the model approach include its inability to simultaneously model the performance of more than one approach at a time. However, roundabouts are typically analyzed on an approach basis and considered to operate as individual yield control points with short distances between these points.

The remainder of the article is organized as follows. In section 2, we review some previous delay models for roundabouts in the UK and Australia. In section 3, we present our formal model description and obtain analytical expressions for the mean and variance of service time. Section 4 describes an empirical study used to evaluate the efficacy of our model when compared with field observations of vehicle delay. In section 5, we provide some concluding remarks and directions for future research.

2. Review of past work

Methodologies have been developed to estimate the capacity of roundabouts and the delay experienced by drivers entering roundabouts in several countries including Great Britain, Australia, Germany, the Netherlands and others. Two primary methodologies exist for the estimation of capacity: empirical models and gap acceptance theory. Empirical regression models were developed in the UK at the Transport and Road Research Laboratory (TRRL) in the late 1970s from data gathered during test track experiments in which full scale models of

roundabouts with varying geometric characteristics operating under saturated conditions were monitored to measure the capacity of individual entries under such conditions (Kimber and Semmens 1977). It was hypothesized that the maximum allowable entry flow for a roundabout is that rate at which vehicles are continuously queued on the subject approach. In such cases, the entry is considered to be saturated and the flow that enters the roundabout during a specified period (typically 1 min, then extrapolated to an hourly rate) is deemed to be the roundabout entry capacity. The TRRL study yielded a linear relationship between the entry capacity and circulating flow rate that accounts for 90% of the variation in the measurement of entry capacity. This relationship is given by

$$q_e = -f_c q_c + F \tag{1}$$

where q_e denotes the entry capacity in veh/h, q_c denotes the circulating stream flow rate across the subject entry in veh/h, f_c is given by the equation

$$f_c = 0.29 + 0.116 w_e \tag{2}$$

and F is given by the equation

$$F = 329 w_e + 35 u + 2.4 D - 135 \tag{3}$$

where w_e denotes the entry width in m, u denotes the circulation width in m and D denotes the size factor in m. All of these factors (w_e, u and D) were found to be significant.

Capacity estimation models using gap acceptance theory have also been developed to estimate the maximum rate at which vehicles on a roundabout (minor) approach can enter the circulating (major) stream of a roundabout. This maximum rate is a function of not only the availability of acceptable gaps in the circulating stream, but also driver behavior which dictates operational characteristics known as critical gap and follow-up time. Critical gap is defined in the HCM as the minimum acceptable gap that entering drivers will utilize to enter the circulating stream (HCM 2000). Follow-up time represents the flow of two or more vehicles from the entering stream into large gaps in the circulating stream. Follow-up time is the headway maintained by minor stream drivers as they enter large gaps in the circulating stream subsequently. With the further assumption that time headways in the circulating stream follow an exponential distribution, Tanner (1967) developed, and Troutbeck (1986, 1988) further refined, the following equation for entry stream capacity at single-lane roundabouts.

$$q_e = \frac{q_c \alpha \exp(-\lambda(t_a - \Delta))}{1 - \exp(-\lambda t_f)} \tag{4}$$

where α denotes the proportion of free vehicles (those vehicles in the circulating stream which are not traveling in platoons), Δ is the headway between platooned vehicles in the circulating stream, λ is given by the equation

$$\lambda = \frac{\alpha q_c}{1 - \Delta q_c}, \tag{5}$$

t_a is the critical gap in s and t_f is the follow-up time also in s. Refinements to equation (4) have been suggested by many researchers to account for the varying critical gap between vehicles in different lanes of a multi-lane roundabout approach and make use of the Cowan M3 distribution (Cowan 1975) for the circulating stream. This brief review of capacity models serves to demonstrate the link between capacity estimation models and delay estimation models.

The average delay, experienced by drivers entering general unsignalized intersections, is now defined in the HCM as the delay experienced accelerating and decelerating to negotiate

entry, time spent in the queue and *service time*, which is defined as the time spent at the head of the queue. Control delay includes 5 s of delay to account for acceleration and deceleration experienced by the driver as he/she travels into and through an unsignalized intersection (Kyte *et al.* 1991) and this adjustment leads to the current unsignalized capacity and delay estimation methodologies included in the HCM 1997, 2000 editions.

Typically, time spent in the queue and service time have been modeled by researchers using M/M/1, M/G/1 or M/G2/1 queuing systems. In most instances, it has been found acceptable to estimate service time, which is an intrinsic part of the delay estimation model, as the reciprocal of entry capacity (Kimber *et al.* 1986, Fisk 1991). In an attempt to account for varying service time, authors have linked service time to the degree of saturation on the subject approach which is defined as the ratio of the entry flow to the capacity on an approach. In particular, Troutbeck (1986) applied the Pollaczek-Khintchine (P-K) formula from the standard results of a M/G/1 queuing system to estimate delay at minor approaches. The delay for an individual driver is given by

$$W = q_e^{-1}(1 + Cx(1 - x)^{-1}) \qquad\qquad (6)$$

where W is the average delay experienced by drivers, $x = q_a/q_e$, is the degree of saturation in which q_a is the approach entry flow rate in veh/h and the constant parameter C is 1.0 for exponential service time and 0.5 for deterministic service time.

Of particular relevance to our model are those approaches utilizing the more general M/G2/1 queuing system which attempts to account for the delay experienced by two types of drivers: (1) those who encounter a busy server upon arrival and (2) those who encounter an idle server (i.e., those who move directly into the server which is defined as the first position of the approach).

One important study which utilizes the M/G2/1 model is by Heidemann and Wegmann (1997) who generalize most previous models and provide analytical results for key measures such as queue length, delay and their associated probability distributions. This article also presents a general capacity formula. The main premise of the work is the use of a general gap–block process in the context of the M/G2/1 queue. In such processes, the gaps between vehicle arrivals in the major stream are random as are the block lengths. The model contributes significantly to the analysis of general unsignalized intersections but requires that gaps are all independent and exponentially distributed random variables. The block (B) is assumed to have a general distribution. Moreover, the arrival process of the subordinate stream is a Poisson process with time-invariant rate. With regard to the gap-acceptance aspects of this model, the authors consider two cases for the behavior of drivers: (i) when the critical gap is assumed to be a fixed value from a known population (consistent drivers) and (ii) when drivers make multiple critical gap choices that may originate from differing populations (inconsistent drivers).

We present a renewal-based analytical approach to compute the mean and variance of the time required for an arbitrary individual in the first position of the approach to enter the circulating stream of a single-lane roundabout. (By arbitrary individual, we mean that we do not distinguish between drivers that see an empty or an occupied first position.) An accurate estimate of the mean and variance of this random time is subsequently applied in a M/G/1 queuing model which uses the P-K formula to determine the overall average number of vehicles waiting to enter a single-lane roundabout (average queue length) and the expected total waiting time (*i.e.* average delay) for an arbitrary driver under stable conditions. Our M/G/1 model can, in fact, be considered as a special case of the one due to Heidemann and Wegmann (1997) wherein the subordinate vehicles arrive to the roundabout according to a Poisson process with fixed intensity (as in the M/G2/1 model) and the circulating traffic stream generates vehicle arrivals according to a renewal process. The latter assumption corresponds to a block length of

zero ($B = 0$) in the approach of Heidemann and Wegmann (1997). It is assumed that all drivers use a fixed critical gap at the approach. We further adopt our renewal-based model to real data from several single-lane roundabout sites in the United States and demonstrate the model's effectiveness for evaluating the average delay. The results of the field experiment indicate good model performance at a variety of flow levels. In section 3, we present the specifics of our mathematical model.

3. Analytical models

In the setting of a single-lane roundabout, vehicles arrive to the approach and wait for an acceptable gap between subsequent vehicles in the conflicting traffic stream before entering the roadway. If an arbitrary vehicle arriving to the roundabout finds a vehicle already in the first position of the approach, then that individual must wait for entry of the first vehicle to the traffic stream before assuming the first position. On the other hand, if a vehicle arrives at the approach and sees no vehicles ahead of it, the driver immediately assumes the first position of the approach and awaits an acceptable gap to enter the circulating traffic stream. In reality, vehicles may not always come to a complete stop in this scenario, but rather, may simply decelerate before proceeding into the circulating stream. Figure 1 graphically depicts a typical roundabout in which the approach and circulating traffic stream are displayed.

This scenario of vehicles arriving to the approach and awaiting entry to the roundabout may be modeled as a queuing system. More specifically, we model the first position of the approach as the *server* in the queuing system and the *queue* is the waiting line of vehicles on the approach seeking 'service' in the first position. If it is assumed that vehicles arrive to the approach according to a Poisson process and that the approach to the roundabout can physically accommodate an infinite number of vehicles, then the system may be modeled as an $M/G/1$ queue. Some of the implications of these assumptions will be discussed subsequently.

The true service time for a vehicle in the first position of the approach includes the time required to wait for an acceptable gap, travel time to enter the circulating stream and the headway for the subsequent circulating vehicle. It should be noted that a point queuing model
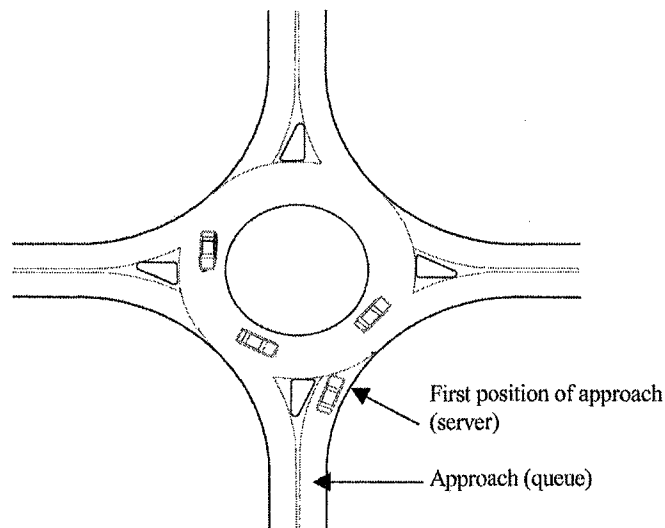

Figure 1. Pictorial representation of a single-lane roundabout.

assumes instantaneous service (*i.e.*, zero elapsed time for vehicle passage into the circulating stream). However, in typical operation modeling, a vehicle is not considered to be 'serviced' until the rear bumper of the vehicle clears the yield bar. Adjustments to our model for this discrepancy are described in section 4.

The total system waiting time is the random time spent on the approach waiting to assume the first position in the queue plus the service time. Applying standard queuing results for the M/G/1 queue (Gross and Harris 1998), the mean number of vehicles in the system (server and queue) in steady state is given by the P-K formula

$$L = \gamma + \frac{\gamma^2 + \lambda^2 \sigma^2}{2(1 - \gamma)}, \qquad (7)$$

from which the mean waiting time for an arbitrary vehicle can be obtained by Little's law as

$$W = L\lambda^{-1} \qquad (8)$$

where λ is the mean arrival rate of vehicles to the queue, μ^{-1} is the mean service time of the server, σ^2 is the variance of the service time and $\gamma \equiv \lambda/\mu$ is the traffic intensity. Clearly, equations (7) and (8) require only the mean arrival rate of vehicles to the approach (λ) and the mean and variance of the service times having a general distribution function, G. However, it is important to note that these equations are valid only under stable operating conditions, *i.e.*, the arrival rate of vehicles to the approach may not exceed the total service rate at the approach. This assumption corresponds to the roundabout system experiencing light to moderate traffic conditions.

In this model, we also assume that the approach to the roundabout is able to accommodate an infinite number of vehicles. Our rationale for doing so is as follows. Let $K < \infty$ denote the total number of vehicles that may be accommodated on the approach (including the first position) for an M/G/1/K queuing system. For K sufficiently large, the M/G/1 queuing results serve as a good approximation to the M/G/1/K system. As it seems practically reasonable to assume that the approach would accommodate at least 10 vehicles and because the steady-state distribution is intractable when the waiting room is finite, we employ the infinite waiting room assumption. In what follows, we review some rudimentary concepts from the theory of renewal processes which are needed to derive our analytical expressions for the mean and variance of the service time.

Let $\{N(t) : t \geq 0\}$ be a counting process such that $N(t)$ denotes the number of occurrences of an event in the time interval $(0, t]$. Define X_n as the time between event $n - 1$ and event n. The counting process $\{N(t) : t \geq 0\}$ is said to be a renewal process if the sequence of non-negative random variables $\{X_1, X_2, \ldots\}$ is independent and identically distributed with arbitrary distribution function F. For example, when the distribution function of the interevent times is exponential, the counting process is said to be a Poisson process.

A delayed process is one in which the distribution function for the first interevent time X_1 is F_e while the sequence $\{X_n: n = 2, 3, \ldots\}$ follows the distribution F. The distribution function F_e, referred to as the equilibrium distribution, is related to F by

$$F_e(t) = \tau^{-1} \int_0^t (1 - F(u)) \, du \qquad (9)$$

where $\tau = E(X_n)$. After the occurrence of the first event, the renewal process is initiated, and henceforth, all interevent times have cumulative distribution function F.

In the case of the single-lane roundabout, the sequence of observations, $\{X_1, X_2, \ldots\}$, correspond to time headways for the circulating traffic stream. In subsection 3.1, we present

the main results of our paper; analytical models to compute the mean and variance of service time as a function of the arbitrary headway distribution F. The derivations of the expressions can be found in Appendix A.

3.1 *Mean and variance of service time*

Let T be a continuous random variable denoting the time for a driver to enter the circulating stream of a single-lane roundabout given that the driver is at the front of the waiting line (*i.e.* T is the service time). Our objective is to calculate the expected value and variance of T given that headway times for the circulating stream follow some general cumulative distribution function denoted by F. The first vehicle attempting to enter the circulating stream arrives at the approach at some intermediate phase of the renewal process and not at the beginning (*i.e.* with probability 1, the first driver does not enter the approach at the inception of a renewal epoch). Hence, the elapsed time until first passage of a circulating stream vehicle will not technically follow the distribution F, but can be assumed to follow the equilibrium distribution (see equation (9)) of F.

Define g as the mean acceptable gap for drivers arriving at the subject approach and τ as the mean time headway for circulating vehicles under the general distribution F. In our context, gaps are considered to be time headways in the circulating stream. It is assumed in this work that all drivers use the same mean acceptable gap (*i.e.* all drivers are consistent and choose the same fixed critical value from the same population). This assumption of our model is somewhat restrictive; however, we use it to gain analytical insight into the behavior of the service time. In the subsequent discussion, it will be seen that the mean and variance of the service time are, in fact, highly dependent on the value of g. We next present the main results of this article and defer the derivations of such to Appendix A. The expected service time may be obtained by conditioning upon the passage time of the first and subsequent vehicles in the circulating stream by the equation

$$E(T) = \frac{1}{\tau}\left(\frac{1}{2}g^2 - \int_0^g tF(t)\,\mathrm{d}t + (1 - F(g))^{-1}\left(g - \int_0^g F(t)\,\mathrm{d}t \int_0^g t\,\mathrm{d}F(t)\right)\right). \quad (10)$$

Derivation of equation (10) is provided in Appendix A.

Using an approach similar to that used for deriving the mean service time, the variance of service time T may also be derived using the fundamental relationship

$$\mathrm{VAR}(T) = E(T^2) - (E(T))^2. \quad (11)$$

Using this relationship, the variance of service time (Appendix A) may be written as

$$\mathrm{VAR}(T) = \frac{1}{\tau}\left(\frac{1}{3}g^3 + E(T_0)g^2 - \int_0^g (t^2 + 2tE(T_0))\cdot F(t)\,\mathrm{d}t\right)$$
$$+ \frac{1}{\tau}\left(g - \int_o^g F(t)\,\mathrm{d}t(1 - F(g))^{-1}\right)\cdot\left(\int_0^g t^2\,\mathrm{d}F(t) + 2E(T_0)\int_0^g t\,\mathrm{d}F(t)\right)$$
$$- [E(T)]^2 \quad (12)$$

where the last term of (12) is obtained by equation (10) and $E(T_0) = (1 - F(g))^{-1}\int_0^g t\,\mathrm{d}F(t)$, (refer equation (A4) in Appendix A). Equations (10) and (12) require only the mean time headway of the circulating stream, the mean acceptable gap and the cumulative distribution function of the interarrival times of the circulating stream. These expressions make no assumptions regarding the time headway distribution; however, if, *e.g.*, the headway distribution is
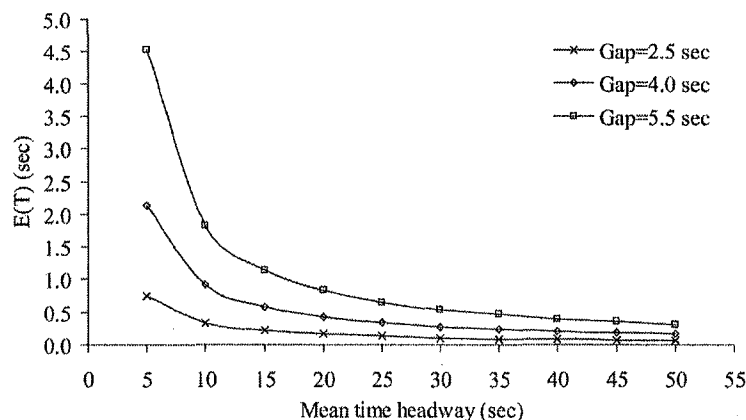
Figure 2.   Expected service time for exponential time headway with various mean acceptable gaps.

exponential with rate θ, then $E(T)$ and VAR(T) are, respectively, given by

$$E(T) = \theta^{-1}(\exp(\theta g) - 1) - g \tag{13}$$

and

$$\text{VAR}(T) = \theta^{-2}((\exp(\theta g) - \theta g)^2 - 1) - g^2. \tag{14}$$

Equation (13) is in agreement with the delay equation originally due to Adams (1936). In order to depict the behavior of $E(T)$, VAR(T) and the effect of the mean acceptable gap, the mean and variance of service time are plotted in figures 2 and 3, respectively, as a function of the mean time headway when the headway distribution of the circulating stream is exponential. In such cases, the coefficient of variation (the ratio of the standard deviation to the mean) of the headway times of the circulating stream is equal to unity.

In agreement with intuition, figure 2 indicates that, for all gap sizes, the expected service time decreases as the circulating stream time headway increases. Moreover, the mean service time increases with the mean acceptable gap. However, the impact of the acceptable gap size diminishes in the region of large time headways as larger gaps permit nearly instantaneous entry to the circulating stream of the roundabout. Figure 3 reveals similar trends for the variance of service time given that the circulating stream headway times are exponentially distributed.
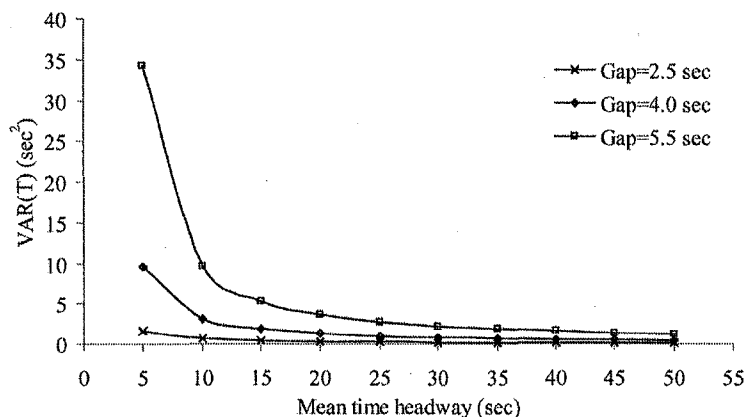

Figure 3.   Variance of service time for exponential time headway with various mean acceptable gaps.
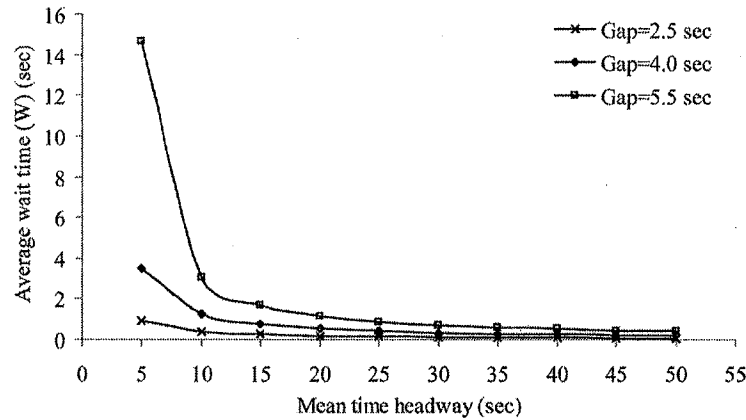
Figure 4. Average wait time for exponential time headway and various mean acceptable gaps (approach arrival rate = 500 veh/h).

Equations (10) and (12) are next utilized in the M/G/1 queuing model to obtain our performance measures, namely the average number of vehicles on the approach given by

$$L = \lambda E(T) + \frac{\lambda^2[(E(T))^2 + \text{VAR}(T)]}{2(1 - \lambda E(T))} \tag{15}$$

and the average waiting time on the approach (queue time plus service time), which is obtained by equation (8). Figure 4 demonstrates the effect of the mean and variance of headway times on the total waiting time (W) for various acceptable gap sizes. In figure 4, headway times are assumed to follow an exponential distribution, and the arrival rate to the approach is 500 veh/h.

The figure indicates that the average waiting time on the approach increases as the mean acceptable gap increases. Intuitively, the same trend is observed as that for the mean and variance of service time, namely that the effect of the gap size diminishes as the mean headway times tend toward infinity.

Equation (15), in conjunction with equation (8), was used to compute the mean queue length (L) and mean waiting time (W) for a number of approach arrival rates when the circulating stream was assumed to be exponentially distributed. Table 1 provides a summary of the performance measures when the mean time headway is exponential with mean 3.0 s (and variance 9.0 s^2) and the mean acceptable gap for drivers entering the circulating stream is 3.5 s.

The derived analytical models of this section require only the mean accepted gap for drivers, the headway distribution of the circulating stream and the arrival rate to the roundabout

Table 1. Performance measures when time headways are exponentially distributed with mean 3.0 s, $g = 3.5$ s.

Approach flow rate (veh/h)	Average number of vehicles in system (L)	Average waiting time (W) (s)
50	0.046	3.314
100	0.098	3.515
200	0.221	3.980
300	0.380	4.553
400	0.587	5.281
500	0.866	6.232
600	1.255	7.530
700	1.823	9.403
800	2.744	12.348

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approach only. The headway distribution of the circulating stream may be approximated by collecting headway data under varying levels of traffic flow. The time headway maintained by a pair of circulating stream vehicles is formally defined as the elapsed time between the front bumper of the first vehicle and the front bumper of the second circulating vehicle with respect to a fixed point of reference. By collecting a sufficiently large number of headway observations at a roundabout site, an empirical distribution function may be obtained. Thereafter, through the use of standard goodness-of-fit tests, it is possible to fit a parametric probability distribution to the headway data. The fitted distribution is then used explicitly in equations (10) and (12) to estimate the mean length of the queue and the average amount of time an arbitrary vehicle waits before entering the circulating stream whenever the system is in steady state (*i.e.*, when traffic conditions are stable). In section 4, we describe a field experiment for single-lane roundabouts in the United States and compare the analytical results to field data for the average delay of vehicles.

4. Data analysis

4.1 *Data collection procedure*

The National Co-operative Highway Research Program (NCHRP) conducted a survey identifying 15 single-lane roundabouts in the United States in 1997 (NCHRP Project 20-5, 1998). Six of these sites are operating in residential or rural areas and carry a low volume of traffic; therefore, six of the remaining nine sites were chosen for analysis in this work.

The data collection sites are shown in table 2 along with information regarding average daily traffic and peak hour volumes. Each site adheres to the basic definition of a roundabout in that it requires entering drivers to yield to circulating traffic and their entries are deflected to slow drivers as they proceed through the roundabout. In addition, each site had been in operation for at least 1 year prior to data collection. Data were recorded at the six single-lane roundabouts by video cameras mounted at each of the roundabout entries and over the circulating roadway for 2 h during the morning and evening peak periods.

In addition to observing traffic volumes, the following operational performance measures were also observed:

- 15-min entering flow rate per approach (veh/h)
- 15-min circulating flow rate per approach (veh/h)
- turning movements (veh/h)
- headway in the circulating stream (s)
- gaps/lags in the circulating stream accepted by entering drivers (s)

Table 2.   Summary of empirical study data collection sites.

Location	Number of approaches	Average daily volume on all approaches (veh/day)	Peak hour volume on all approaches (veh/h)
Palm Beach County, FL	4	7,600	510
Lisbon, MD	4	8,500	856
Tallahassee, FL	3	17,825	1085
Fort Walton Beach, FL	3	12,000	1245
Lothian, MD	4	15,000	1345
Boca Raton, FL	4	16,000	1450

- gaps/lags in the circulating stream rejected by entering drivers (s)
- follow-up time maintained by two consecutively entering vehicles (s)
- service time (s)
- time spent in the queue (s).

It should be noted that the queue lengths were not explicitly observed and recorded in this experiment. The required inputs for evaluating the efficacy of equations (10) and (12) were the average accepted gap by drivers, the mean and variance of time headways for the circulating stream and the distribution of headway in the circulating stream. The two former measures were observed directly while the latter must be approximated.

4.2 Determination of headway distribution

Microscopic circulating stream headway data were collected for each roundabout entry under varying traffic flows. Headway observations for 141, 15-min periods were tested against parametric cumulative distribution functions at several circulating stream flow rates in addition to one aggregate test. The headway data were grouped and tested as shown in table 3.

Headway measurements were partitioned into 80 categories ranging from 0 to 200 s in 2.5 s increments. Empirical distribution functions were computed for each of the six hourly flow rates and a software package (BestFit) was used to determine which parametric distribution best represented the empirical time headways by performing both Chi-square and Kolmogorov–Smirnov (K–S) goodness-of-fit tests. In total, twenty-one parametric distributions including lognormal, exponential, gamma, Weibull and normal were considered. Our experiments indicate that the lognormal distribution is best suited to approximate the empirical data given its consistent ranking of 1 or 2 to represent the distribution of headways under a variety of flow conditions. The results of the goodness-of-fit tests for these distributions are given in table 4 for each of the flow ranges. All tests were performed at the 0.01 level.

Table 3. Circulating stream headway data measurements.

Circulating stream flow (veh/h)	Number of operational periods	Number of headway measurements
0–200	81	2406
201–400	38	1893
401–600	15	1106
601–800	6	849
801–880	1	222
0–880	141	6476

Table 4. Goodness-of-fit results for lognormal and exponential headway distribution.

Flow rate (veh/h)	Lognormal			Exponential		
	Bestfit rank	Chi-square statistic	K–S statistic	Bestfit rank	Chi-square statistic	K–S statistic
0–200	1	0.287	0.183	4	1.28	0.151*
201–400	1	0.258	0.043	6		0.177*
401–600	2	0.190	0.047	7	1179.53*	0.210*
601–800	2	0.068	0.047	7		0.115
801–880	1	0.107	0.039	6	449.10*	0.159
0–880	1	0.220	0.050	9	10.121	0.217*

*Denotes a statistically significant test at the 0.01 level.

Table 5. Lognormal versus exponential headways, assumed $g = 4.0$ s.

Mean headway (s)	$E(T)$ (s)		VAR(T) (s^2)	
	Lognormal	Exponential	Lognormal	Exponential
5	2.57	2.13	12.95	9.50
10	1.01	0.92	3.46	3.17
15	0.61	0.58	1.80	1.85
20	0.43	0.43	1.19	1.30
25	0.34	0.34	0.90	1.0
30	0.28	0.28	0.72	0.81
35	0.23	0.24	0.60	0.68
40	0.20	0.21	0.52	0.59
45	0.18	0.18	0.46	0.52
50	0.16	0.16	0.41	0.46

Table 4 indicates that the empirical distribution function of time headway was not found to be statistically significantly different from the lognormal distribution in any of the six cases using both goodness-of-fit tests. Moreover, the software package ranked the lognormal as the first or second best fit in each case. In contrast to these results, the data indicate a statistically significant difference from an exponential distribution for all but one of the cases. Traditionally, deterministic models widely published and used by the transportation community utilize the simplistic exponential distribution for modeling headways. To better understand the implications of this assumption, we evaluate our analytical model using both lognormal and exponential headway distributions. Equations (10) and (12) were numerically computed using MATLAB© codes running on a personal computer. We first allowed the coefficient of variation of the circulating stream to equal unity in each case and simply varied the mean headway time. The mean acceptable gap was assumed to be 4.0 s. Table 5 demonstrates that if the circulating stream headway distribution has coefficient of variation equal to one, then the difference between using the lognormal and exponential distributions is small for both the mean and variance of service time. As is demonstrated when headways begin to become less random, the two estimations of expected service time begin to diverge. This may suggest that the lognormal distribution is a better representation of actual operating conditions under high flow conditions.

Next, we examine the impact of the coefficient of variation on the estimation of the mean and variance of service time. Figures 5 and 6 demonstrate that there is very little difference
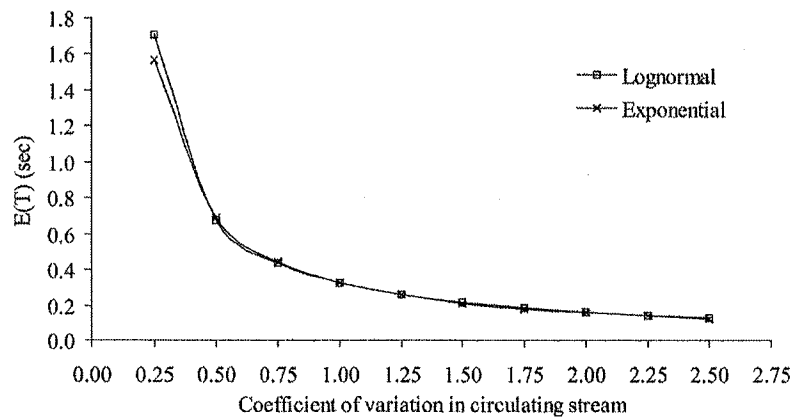

Figure 5. Expected service time as a function of coeficient of variation of the circulating headway distribution.
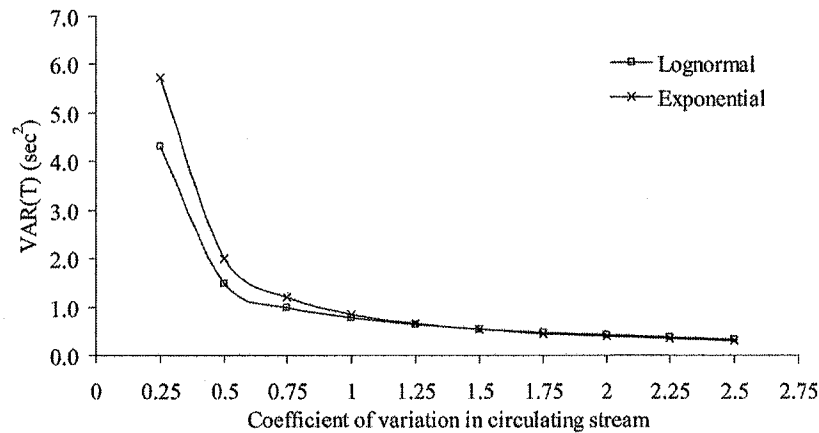
Figure 6. Variance of service time as a function of coefficient of variation of the circulating headway distribution.

between the two approximations. However, the two curves can be distinguished more easily in the case of the variance of service time.

For the purpose of illustration, we computed the average wait time in the system for a vehicle arriving to the approach assuming that the headway distribution of the circulating stream is lognormal and exponential. The mean gap size was set to 3.5 s and the approach arrival rate was 500 veh/h. Figure 7 demonstrates that it is difficult to distinguish between the two curves.

In subsection 4.3, we compare average delay values obtained from our analytical models to field observations taken from the various roundabout sites. The empirical experiment serves to validate our renewal theory-based approach for modeling the service time and overall delay.

4.3 *Comparisons with field data*

Numerical values via equations (10) and (12) were compared with field measurements of the average delay experienced by drivers. We do not compare the average queue length, as this performance measure was not explicitly observed in the field experiment. It is important to note that a point queuing model assumes zero elapsed time for vehicle passage into the
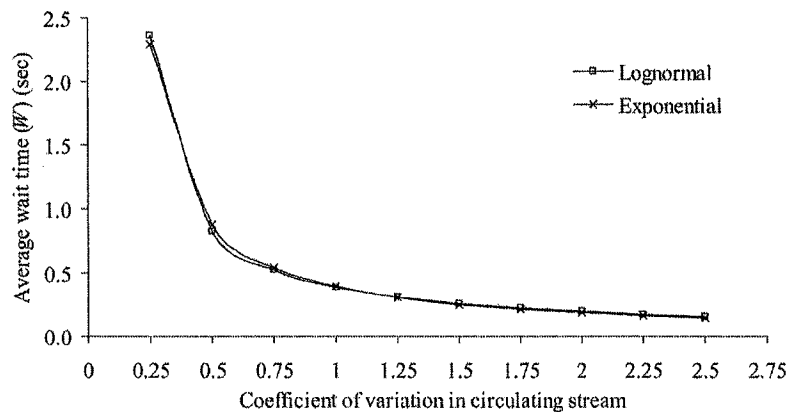

Figure 7. Average wait time as a function of coefficient of variation of the circulating headway distribution ($g = 3.5$ s, approach volume $= 500$ veh/h).

Table 6.   Comparison of analytical versus empirical delay values at various flow ranges.

Approach volume (veh/h)	Mean circulating headway (s)	Variance circulating headway (s^2)	Average acceptable gap (s)	Measured delay (s)	Average delay (s)	
					Lognormal	Exponential
12	8.0	100.0	5.5	2.0	3.92	3.44
24	5.0	39.0	5.2	7.0	6.10	5.20
36	9.0	128.0	6.0	1.0	4.22	3.68
44	9.0	115.0	8.0	6.0	7.71	6.44
68	5.0	22.0	4.9	7.0	5.85	4.88
68	9.0	139.0	7.4	5.0	6.82	5.69
348	35.0	939.0	5.9	2.0	1.80	1.83
408	15.0	249.0	5.5	4.0	3.17	2.98
420	16.0	233.0	5.4	2.0	2.88	2.78
584	26.0	616.0	5.3	2.0	2.21	2.22

circulating stream. However, when service times were measured in the field, a vehicle was not considered serviced until the rear bumper of the vehicle had cleared the yield bar, which is typical in operations modeling. In order to adjust for this discrepancy between the model and field measurements, the average vehicle passage time (found to be ~1.0 s from several time periods and sites) was added to the model estimate of mean service time of equation (10).

Table 6 gives a summary of the delay values computed using our models as compared to the values observed at various sites in the United States. Only a sample of the available data were extracted because of time and budget limitations of the study. Table 6 contains a range of conditions from low to high entering flow and from low to high circulating headways. Each row represents a particular roundabout approach operating under the conditions described for a 15-min study period. As the proposed models are based exclusively on queuing theory and do not include any potential influences of geometric parameters, it is acceptable to combine the results as shown in table 6. To better understand the implications of traditional models that assume an exponential distribution of headways, we computed the expected delay for two cases: (1) when the circulating time headways are assumed to follow an exponential distribution and (2) when they are assumed to follow a lognormal distribution. In table 6, the field observed average delay are shown in column 5, and the estimated delay are shown in columns 7 and 8.

The discrepancies between the model and field results may be attributed to a few different factors. First, our model assumes that the roundabout approach behaves exactly as a queuing system in which vehicles move out of queue and into service in a stop–go fashion. In reality, if drivers observe an acceptable gap while approaching the first position, they tend to simply decelerate and then proceed directly into the circulating stream without stopping. This behavior is not captured in our model. Moreover, the model assumes that arrivals occur to the approach according to the Poisson process with a time-invariant mean arrival rate. This may not be the case in reality, even for a short observation period of 15 min.

5.   Conclusions

This paper provides explicit, renewal theory-based analytical models to estimate the mean and variance of service time for a driver in the first position of a single-lane roundabout approach that is able to accommodate any distribution of time headways in the circulating stream whatsoever. The model can easily compute the performance metrics on a personal computer. These analytical models can subsequently be applied in an M/G/1 queuing model

of the roundabout approach to compute the desired performance measures, namely the average delay experienced by an arbitrary driver arriving at the approach attempting to enter the circulating stream. To our knowledge, this general model of the mean and variance of service time is the first of its kind in that it explicitly models the service time apart from estimates of capacity. Results obtained using the analytical technique were compared against actual field measurements of the mean and variance of service time and the overall average delay experienced by drivers. Our model was shown to perform well in light to moderate traffic conditions, the scenario for which we expect it to perform well. In saturated traffic conditions, the model is not expected to perform as well.

The field study further contributes to a better understanding of operational characteristics of single-lane roundabouts in the United States. The data collected from six single-lane roundabout sites indicated that headway times of the circulating stream are most closely approximated by the lognormal distribution. However, in computing the overall average delay, there was not much benefit gained by using the lognormal when compared with assuming that the circulating stream time headways are exponential. For this reason, it may be possible (and computationally more expedient) to use the simpler closed-form analytical expressions of equations (13) and (14).

The use of roundabouts as an alternative to standard intersections is increasing rapidly in the United States. To date, many of the analysis tools utilized by transportation professionals to study roundabout performance are based on studies and models developed outside of the United States. As driver behavior is a key factor to roundabout performance, ideally empirically based studies will be conducted to better represent driver behavior in the United States. As the number of installed roundabouts increases, more data will become available to help refine and improve the models proposed herein. Hence, a number of future research directions may be identified. First, this work assumed that all drivers use the same mean acceptable gap for entering the circulating stream. It will be instructive to investigate the more generalized gap acceptance behavior such as those considered by Heidemann and Wegmann (1997). Another extension is to model the components of service time that were assumed fixed in our preliminary model (*i.e.* car lengths, travel time into the stream and acceleration/deceleration) as a random component. Finally, this work serves as a stepping stone for the analysis of multi-lane roundabouts. With over 300 documented roundabouts in the United States and approximately one-third of them operating in a variety of multi-lane configurations, there is a need to model the operational performance of multi-lane roundabouts based on US conditions. The complexity of the multi-lane problem appears, at first glance, to be significantly greater owing to weaving patterns in the circulating roadway and gap acceptance behavior of entering drivers. Thus, an analysis of the headway distribution of the circulating traffic stream and the gap acceptance behavior of simultaneously entering vehicles is needed.

References

Adams, W.F., Road traffic considered as a random series. *J. ICE*, 1936, **4**(1), 121–130.
Cowan, R.J., Useful headway models. *Transport. Res.*, 1975, **9**(6), 371–375.
Fisk, C.S., Traffic performance analysis at roundabouts. *Transport. Res.*, 1991, **25B**(2/3), 89–102.
FDOT, Florida Department of Transportation, *Florida Roundabout Guide*, (1995).
Gross, D. and Harris, C.M., *Fundamentals of Queueing Theory* (3rd Edn), 1998 (John Wiley & Sons: New York).
Heidemann, D. and Wegmann, H., Queueing at unsignalized intersections. *Transport. Res.*, 1997, **31B**(3), 239–263.
Myers, E.J., Modern roundabouts for Maryland. *ITE J.* October, 1994, 18–22.
Kimber, R.M. and Semmens, M.C., A track experiment on the entry capacities of offside priority roundabouts. TRRL Laboratory Supplementary Report 334, Transport and Road Research Laboratory, Crowthorne, 1977.
Kimber, R.M., Summersgill, I. and Burrow, I.J., Delay processes at unsignalized junctions: The interrelation between queueing and geometric delay. *Transpn Res.*, 1986, **20B**(6), 457–476.
Kyte, M., Zegeer, J. and Lall K., Empirical models for estimating capacity and delay at stop-controlled intersections in the United States. *Intersections Without Traffic Signals II*, 1991 (Springer-Verlag: New York).

MdSHA, State of Maryland Department of Transportation, State Highway Administration, *Roundabout Design Guidelines*, 1995.

Tanner, J.C., The capacity of an uncontrolled intersection. *Biometrica*, 1967, **54**(3), 657–658.

Transportation Research Board, National Research Council, Highway Capacity Manual. Special Report 209, Washington, DC, 2000.

NCHRP 20-5, Transportation Research Board, National Research Council, Modern Roundabout Practice in the United States, Washington, DC, 1998.

Troutbeck, R.J., Average delay at an unsignalized intersection with two major streams each having a dichotomised headway distribution. *Transport. Sci.*, 1986, **20**(4), 272–86.

Troutbeck, R.J., Current and future Australian practices for the design of unsignalized intersections. *Intersections Without Traffic Signals*, 1988 (Springer: Berlin).

Appendix A: derivation of main results

The following derivations are based on a conditioning/unconditioning argument commonly used in stochastic modeling. Some definitions are first presented.

Define

$F(\cdot)$, the distribution function of headway times for the circulating stream; $F_e(\cdot)$, the equilibrium distribution associated with $F(\cdot)$; g, the mean accepted gap for drivers seeking entry to the circulating stream; τ, the mean headway time of the circulating stream; T, the total random time required for service for vehicle in first position of approach; T_0, the total random time required for service assuming a non-delayed renewal process; T_1, the random time to first circulating stream arrival under the equilibrium process; T_2 the random time to first circulating stream arrival under the non-delayed renewal process.

Derivation of equation (10)

If the first gap experienced by the approach driver is less than an acceptable gap (g), the expected service time is the time that the driver has already waited plus the expected time under the general renewal process, $E(T_0)$. Using a standard conditional expectation approach, we see that

$$E(T|T_1 = t_1) = \begin{cases} t_1 + E(T_0) & t_1 \leq g \\ 0 & t_1 > g \end{cases} \tag{A1}$$

i.e., if the first gap experienced upon arrival to the approach is greater than the driver's acceptable gap (g), then the driver experiences zero delay. However, if this first gap is unacceptable, the driver must wait for the first arrival to pass and then it may be assumed that the renewal process follows the distribution $F(\cdot)$ from this point forward. Hence, the total wait is $t_1 + E(T_0)$. An expression for $E(T_0)$ is derived in a similar manner by conditioning on the first arrival of the non-delayed renewal process.

$$E(T_0|T_2 = t_2) = \begin{cases} t_2 + E(T_0) & t_2 \leq g \\ 0 & t_2 > g \end{cases} \tag{A2}$$

Unconditioning (A1) and (A2), we obtain the following results:

$$E(T) = E(E(T|T_1)) = \int_0^g E(T|T_1 = t_1)\, dF_e(t_1) + \int_g^\infty 0\, dF_e(t_1)$$

$$= \int_0^g t_1\, dF_e(t_1) + E(T_0)F_e(g) \tag{A3}$$

and

$$E(T_0) = E(E(T_0|T_2)) = \int_0^g (t_2 + E(T_0)) \cdot dF(t_2) + \int_g^\infty 0 \cdot dF(t_2)$$

$$= (1 - F(g))^{-1} \int_0^g t_2 \, dF(t_2) \tag{A4}$$

Substituting equations (A4) into (A3) and using the relationship between the equilibrium distribution and the interarrival distribution yields the mean service time for a driver in the first position of the approach.

$$E(T) = \frac{1}{\tau} \left(\frac{1}{2} g^2 - \int_0^g t F(t) \, dt + (1 - F(g))^{-1} \left(g - \int_0^g F(t) \, dt \int_0^g t \, dF(t) \right) \right) \tag{A5}$$

Derivation of equation (12)

By definition,

$$T = T_0 + T_1 \tag{A6}$$

which implies

$$E(T^2|T_1 = t_1) = \begin{cases} t_1^2 + 2t_1 E(T_0) + E(T_0^2), & t_1 \le g \\ 0, & t_1 > g \end{cases} \tag{A7}$$

and

$$E(T_0^2|T_2 = t_2) = \begin{cases} t_2^2 + 2t_2 E(T_0) + E(T_0^2), & t_2 \le g \\ 0, & t_2 > g \end{cases} \tag{A8}$$

Unconditioning (A7) and (A8), we obtain the following result.

$$E(T^2) = \int_0^g (t_1^2 + 2t_1 E(T_0)) \, dF_e(t_1) + \int_0^g E(T_0^2) \, dF_e(t_1)$$

$$= \int_0^g (t_1^2 + 2t_1 E(T_0)) \, dF_e(t_1) + E(T_0^2) F_e(g) \tag{A9}$$

and

$$E(T_0^2) = \int_0^g (t_2^2 + 2t_2 E(T_0)) \, dF(t_2) + \int_0^g E(T_0^2) \, dF(t_2)$$

$$= (1 - F(g))^{-1} \cdot \int_0^g (t_2^2 + 2t_2 E(T_0)) \, dF(t_2)) \tag{A10}$$

Substituting equations (A10) into (A9), we obtain

$$E(T^2) = \int_0^g (t_1^2 + 2t_1 E(T_0)) \, dF_e(t_1) + F_e(g)(1 - F(g))^{-1} \cdot \int_0^g (t_2^2 + 2t_2 E(T_0)) \, dF(t_2)$$

which, with simplification yields

$$E(T^2) = \frac{1}{\tau} \left(\frac{1}{3} g^3 + E(T_0)g^2 - \int_0^g (t^2 + 2t E(T_0)) \cdot F(t) \, dt \right)$$

$$+ \frac{1}{\tau} \left(g - \int_0^g F(t) \, dt (1 - F(g))^{-1} \right) \cdot \left(\int_0^g t^2 \, dF(t) + 2E(T_0) \int_0^g t \, dF(t) \right) \tag{A11}$$

Hence, our final result is obtained.

$$
\begin{aligned}
\mathrm{VAR}(T) = {} & \frac{1}{\tau} \left(\frac{1}{3} g^3 + E(T_0) g^2 - \int_0^g (t^2 + 2t E(T_0)) \cdot F(t) \, \mathrm{d}t \right) \\
& + \frac{1}{\tau} \left(g - \int_0^g F(t) \, \mathrm{d}t (1 - F(g))^{-1} \right) \cdot \left(\int_0^g t^2 \, \mathrm{d}F(t) \right. \\
& \left. + 2E(T_0) \int_0^g t \, \mathrm{d}F(t) \right) - [E(T)]^2
\end{aligned}
\tag{A12}
$$

where $E(T_0)$ is obtained by equation (A4) and the last term of equation (A12) is obtained by equation (A5).